# Comparison of methods for crack tip opening displacement toughness determination

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The crack tip opening displacement (CTOD) initiation toughness has been obtained by different methods in Cr and Cr–Mo steels at 30, 200 and 400 °C. The crack tip stretching and grain deformation has been investigated by scanning electron microscope and optical microscopic studies and by microhardness measurements. The resistance curve approach is used employing the average and the maximum crack growth and the initiation toughness determined are with respect to 0.2 mm crack growth ( $\delta^{0.2}$ ), the stretched zone width (SZW) and also using a blunting line approach. In addition, an initiation toughness using stretched zone depth (SZD) measurement is also obtained. The various initiation toughness values have been compared and an attempt has been made to identify the realistic plane strain CTOD toughness amongst the different values. The  $\mathcal{J}$ -CTOD relationship has also been investigated.

## 1. Introduction

The crack tip opening displacement (CTOD) is used as the elastic-plastic toughness parameter and hence also for providing inspection standard to structures where fracture or failure is preceded by ductile crack growth. The standard for CTOD ( $\delta$ ) measurement is provided by BS 5762 [1] using a clip gauge method in bending geometry. The method involves use of rotation factor (r) to convert the crack mouth opening displacement (CMOD) into the CTOD. The rotation factor is standardized only for the three-point bend geometry and some empirical approaches have to be used to determine the rotation factor for other geometries [2, 3]. In addition, the rotation factor changes with a change in the ratio of plastic to elastic component of displacement as well as with the stable crack growth [4, 5]. Contradictory reports have been made by various investigators with regard to the numerical value of rotational factor in different geometries and the method used for its evaluation. Pratap and Pandey have even suggested a method to evaluate CTOD without the use of the rotational factor [6]. Kolednik, recently [7] suggested a value of r = 0.96 in place of 0.4 as recommended by BS 5762 leading to a further uncertainty.

Besides the rotation factor, another point of controversy is the definition of the crack initiation CTOD. The latter may be defined by various ways e.g. (a) CTOD corresponding to nil physical crack growth, (b) CTOD corresponding to point of intersection of  $\delta$  resistance curve with the blunting line analogus to J toughness determination [8]. (c) CTOD corresponding to a specified amount of crack growth (2%, 0.2 mm etc.) on the  $\delta$ -R curve and (d) CTOD at the critical stretched zone width (SZW). The ASTM Committee on Fracture testing [9] has also in fact

ding with regard to the crack initiation toughness in ductile alloys, the present investigation has been undertaken. A considerable difference may be expected between and-the initiation toughness  $(\delta_i)$  values obtained by differand ent approaches leading to an unrealistically conservative value on one hand and a too optimistic value

toughness, CTOD.

on the other hand. The different methods for  $\delta_i$  evaluation hence need to be compared and the values of  $\delta_i$  need to be checked against a more direct method based on the microscopic measurements near the crack tip. The present investigation has been conducted with the above objectives in mind.

suggested finding the initiation CTOD for the 0.2 mm

crack growth on  $\delta - R$  curve. The European Group on

Fractures (EGF) [10] has on the other hand, proposed

a different methodology to evaluate the initiation

In view of the prevailing multiplicity of opinions

# 2. Experimental procedure

### 2.1. Materials and test specimens

The investigation was carried out using two structural low alloy steels, i.e. a  $2\frac{1}{4}$  Cr-1 Mo steel and a  $\frac{1}{2}$  Mo steel. The chemical composition and tensile properties of the alloys are shown in Table I.

The single-edge notched (SEN) bend specimens were machined as per ASTM standard with thickness, B = 11 mm, width, W = 22 mm and crack depth to width ratio, a/W = 0.5. Fatigue precracking was provided using standard method [1].

# 2.2. Fracture mechanics test

The specimens were tested on an Instron machine of 5 tonne capacity under a three-point bend loading. The

TABLE I Chemical composition (wt %) and tensile properties of alloys

Alloy	С	Mn	Si	S	Р	Cr	Мо
Mo steel Cr–Mo steel	0.21 0.14	0.81 0.48	0.255 0.330	0.023 0.026	0.034 0.035	0.2 2.0	22 0.45 07 0.86
Alloy	Testing temperature (°C)	Yield stro σ <sub>y</sub> (MPa)	ength )	Tensile strength σ <sub>u</sub> (MPa)	Elongation (%)	RA (%)	Strain hardening exponent, n
	30	349		540	33	68	0.190
Mo steel	200	294		532	25	60	0.212
	400	306		579	28	56	0.203
	30	389		566	33	78	0.170
Cr-Mo steel	200	337		503	30	79	0.203
	400	331		515	26	76	0.186

CMOD was measured using a clip gauge and output was taken on a d.c. microvoltmeter. The load point displacement (LPD) was measured with a dial gauge. The CMOD and LPD readings were taken at regular intervals with load. The tests were conducted at ambient temperatures (30.°C), 200 and 400 °C. For other than ambient temperature tests, two heating pads consisting of heating coils of 200 $\Omega$  resistance were placed on either sides of the specimen. Electrical insulation was provided by using mica sheets between the heating pad and the specimen surface. Current was passed through the heating coil by regulated power supply and the temperature was measured by a thermocouple spot welded near the crack.

The loading was continued to an appropriate level in the elastic-plastic region to allow some crack growth in the specimen. Four to six identical specimens were loaded to give varying amounts of crack growth. The actual amount of crack growth was measured after break opening the specimen in liquid nitrogen. The average crack growth ( $\Delta a_{av}$ ) using the nine-point average method and the mid thickness (maximum) crack growth ( $\Delta a_{max}$ ) were measured on the broken surfaces.

#### 2.3. Microscopic studies of stretched-region

For studying the profile of stretched zone (SZ) specimens were sectioned at the mid thickness. The surface was polished and mounted vertically so that the mid thickness plane comes under the SEM beam (Fig. 1). The back tilt was varied between 10 and  $30^{\circ}$  to make the profile clear. The stretched zone width (SZW) as



Figure 1 Schematic diagram of SZD measurement B thickness, W width of specimen.

well as the depth (SZD) were measured. The details are reported elsewhere [11].

The transverse section of the fracture in the vicinity of stretched zone was investigated using an optical microscope to reveal grain deformation and structural features. The microhardness measurements were also taken across the SZ on the transverse section to ascertain grain deformation characteristics.

#### 3. Results

3.1. Determination of CTOD and

CTOD-resistance ( $\delta$ --R) curves

To obtain the CTOD at the original crack tip, the change in rotation centre due to physical crack growth was taken into account resulting in an expression for CTOD [11]

$$\delta = \delta_{e} + \delta_{p} = \frac{K^{2}(1-v^{2})}{2\sigma_{y}E} + V_{gp} \left(\frac{a+\Delta a+z+r(W-a-\Delta a)}{r(W-a-\Delta a)}\right)^{-1}$$
(1)

where  $\delta_e$  and  $\delta_p$  are the elastic and plastic components of CTOD, K the stress intensity factor corresponding to the critical load,  $\sigma_y$ , E and v the yield strength, elastic modulus and Poisson's ratio, respectively,  $V_{gp}$ the plastic component of CMOD and r the rotation factor. A value of r = 0.4 was used along the line of BS 5762 recommendations. The values of  $\delta$  were computed at different amount of crack growth by substituting the average as well as the maximum crack growth separately in the above equation and termed as  $\delta_{av}$  and  $\delta_{max}$ , respectively. It was noted that the proposed BS 5762 method to find CTOD (without considering a change in rotation centre) resulted in a much more conservative value and was hence discarded in favour of Equation 1.

In attempting to draw  $\delta - R$  curves, it was noticed that the best fit curves were either linear or non-linear with second-degree polynomial. This is contrary to the recommendations by BS 5762 or ASTM E 813-88 where a linear regression line is suggested for the best fit for the  $\delta - R$  and J - R curves respectively. All the R curves were analysed by using both the linear and the non-linear regression analysis and the best fit was used

TABL	ΞII	Best	fit equations	for	δ–R	curves	(δ	in	μm,	Δa i	n m	ım)
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Alloy	Test temperature (°C)	$\delta_{av}$ against $\Delta a_{av}$	$\delta_{\max}$ against $\Delta a_{\max}$
		$\delta_{av} =$	$\delta_{max} =$
	30	$289 + 173\Delta a + 77\Delta a^2$	$319 + 4.6\Delta a + 86\Delta a^2$
Mo steel	200	$228 + 33.3\Delta a + 84.5\Delta a^2$	$49 - 35.7\Delta a + 59.6\Delta a^2$
	400	$82 + 419\Delta a + 11.9\Delta a^2$	$96 + 202\Delta a + 19.5\Delta a^2$
	30	$158 + 834.3\Delta a$	$75 + 579\Delta a$
Cr-Mo steel	200	$285 + 696.5\Delta a$	$151 + 555.7\Delta a$
	400	$237 + 414.1\Delta a$	$191 + 287\Delta a$

TABLE III Description of crack tip stretched zone

Alloy		Test temperature (°C)	SZW (μm)	SZD (µm)	Blunting angle, $\theta$ (deg)	d (µm)	Degree of straining VHN
Mo Steel		30	170	137	39	360	35
		200	130	115	41	280	24
		400	100	55	29	230	19
	HAZ	30	129	121	43	_	-
Cr-Mo steel		30	110	58	28	333	46
		200	~140	101	36	392	52
		400	170	111	33	450	59
	HAZ	30	91	69	37	—	-



Figure 2 Typical  $\delta$ -resistance curves (a) Mo steel, 30 °C (b) Cr-Mo steel, 200 °C (c) Cr-Mo steel, 400 °C ( $\Box \ \delta_{av} - \Delta a_{av}, \Delta \ \delta_{max} - \Delta a_{max}$ ).

as the representative R curve. The R curves were obtained using the average as well as the maximum crack growth,  $\delta_{av}$  against  $\Delta a_{av}$ , and  $\delta_{max}$  against  $\Delta a_{max}$ . Typical R curves are presented in Fig. 2. The best fit equations for  $\delta - R$  curves are reported in Table II.

#### 3.2. Stretched zone studies

The stretched zone width, stretched zone depth and blunting angle  $\theta$  (calculated as tan<sup>-1</sup> (SZD/SZW) results are given in Table III. Table III also includes some data on stretched zone measurements taken from the heat affected zone (HAZ) of the alloys the details of which are reported elsewhere [11]. It is seen from Table III that the blunting angle varies between 28 and 43° and is thus smaller than the conventionally assumed angle of 45°.

The microscopic study of the transverse section of the fracture surface from the stretched region revealed an elongated grain structure (Fig. 3). The length of the



*Figure 3* Schematic representation of elongated grain structure near stretched zone.

deformed region, d, consisting of elongated ferrite grains was measured along the crack propagation direction and reported in Table III. The length, d, is found to decrease with increasing temperature for the



Figure 4 Microhardness distribution in ferrite region (a) Mo steel, (b) Cr-Mo steel.

Mo steel and increase with temperature in Cr–Mo steel. The above trend is similar to the variation of SZW and SZD with temperature in these alloys.

The results on microhardness distribution in ferritic region are shown in Fig. 4. The maximum hardness value was noted at the crack surface which decreased to the minimum (base) level near the undeformed grains. The degree of straining (i.e. maximum hardness-minimum hardness) is found to decrease with increasing temperature in Mo steel and increase with temperature in the Cr-Mo steel (Table III).

#### 4. Discussion

# 4.1. Different types of crack initiation toughness

The crack initiation toughness,  $\delta_i$  may be determined in the following four ways.

(i) As the y intercept of the  $\delta - R$  curve to give CTOD at zero crack growth.

(ii) Intersection of blunting line with R curve. The equation of blunting line may be expressed as,

$$\delta = 2\Delta a \tag{2}$$

which is deduced from [8].

(iii) CTOD at the intersection of 0.2 mm physical crack growth with R curve as suggested in [9].

(iv) CTOD at the intersection of (critical) SZW on the R curve.

The above approaches to evaluate  $\delta_i$  are shown in Fig. 5. The  $\delta_i$  values are presented in Table IV. The  $\delta^0$ ,  $\delta^B$ ,  $\delta^{0.2}$  and  $\delta^s$  represent the  $\delta_i$  values at zero crack growth, blunting line intersection, 0.2 mm crack growth and SZW intersection, respectively. The subscripts 'av', and max' indicate the  $\delta_i$  values based on  $\delta_{av}$  against  $\Delta a_{av}$  and the  $\delta_{max}$  against  $\Delta a_{max} R$  curves, respectively.

In Cr-Mo steel, the initiation toughness values based on  $\delta_{max}$  against  $\Delta a_{max} R$  curves are found to be smaller as compared to the toughness values based on  $\delta_{av}$  against  $\Delta a_{av} R$  curves. At the mid-thickness section, the stable cracking is initiated earlier and its propagation is faster. A small element at the midsection is



Figure 5 Typical diagram showing  $\delta_i$  evaluation (Cr–Mo steel, 200 °C) ( $\Box \ \delta_{av} - \Delta a_{av}, \ \Delta \ \delta_{max} - \Delta a_{max}$ ).

believed to nearly satisfy a plane strain condition and a resistance curve based on the mid-section crack growth (i.e.  $\delta_{\max}$  against  $\Delta a_{\max} R$  curve) represents a conservative and realistic situation. In Mo steel, however, the  $\delta_i$  values based on the  $\delta_{\max}$  against  $\Delta a_{\max} R$ curves are not always conservative as compared to the  $\delta_{av}$  against  $\Delta a_{av} R$  curves. The nature of the R curve in the Mo steel is non-linear, i.e. obtained by employing a second-order polynomial regression whereas the R curve is of a linear nature in the Cr-Mo steel. This is apparently the reason for a difference in the behaviour of  $\delta_i$  for the Mo steel.

Except for a few cases, the  $\delta^0$  values are quite conservative as compared to  $\delta^B$ ,  $\delta^{0.2}$  and  $\delta^s$  values.  $\delta^0$ may thus provide a lower limit of initiation toughness and, therefore, may not be a true representative of the actual crack growth resistance of material. Amongst  $\delta^B$ ,  $\delta^{0.2}$  and  $\delta^s$ , the  $\delta^{0.2}$  generally appear to provide a crack initiation toughness on the high side leading to an overestimation of  $\delta_i$  values. The  $\delta^B$  and  $\delta^s$  values are fairly comparable.

TABLE IV Crack initiation toughness values in alloys (in µm)

Alloy	Test temperature (°C)	$\delta^0$		$\delta^{B}$	$\delta^{B}$		$\delta^{0.2}$		$\delta^s$	
		$\delta^0_{av}$	$\delta_{max}^0$	$\delta^{B}_{av}$	$\delta^{B}_{max}$	$\delta^{0.2}_{av}$	$\delta_{max}^{0.2}$	$\delta^s_{av}$	$\delta_{max}^{s}$	
Mo steel	30	289	318	318	318	325	320	320	320	
	200	228	249	235	249	240	249	228	249	
	400	82	96	120	110	170	145	125	120	
Cr–Mo steel	30	157	75	270	110	325	191	249	140	
	200	285	151	440	210	424	262	382	228	
	400	237	191	300	220	320	248	307	240	

TABLE V A comparison of plane strain initiation toughness

Alloy	Test	$\boldsymbol{\delta}_{e}$	$\delta_{Ic}$ ( $\delta + 2SZD$ )	$\delta_{\text{max}}^{s}$	$\delta^B_{max}$
	(°C)	(µm)	(μm)	(µm)	(µm)
Mo steel	30	25	299	320	318
	200	26	256	249	249
	400	29	139	120	110
HAZ	30	34	276	280	295
Cr–Mo	30	31	147	140	110
steel	200	33	230	228	210
	400	44	266	240	220
HAZ	30	21	159	140	135

# 4.2. Stretched zone method for CTOD toughness

From Table III it can be seen that the SZD values are invariably smaller than the corresponding SZW values. It is our contention that 2 SZD (at crack initiation) is a more appropriate measure of crack initiation CTOD. It does not, however, include the elastic component of CTOD. The actual crack initiation CTOD would, therefore, be given by

$$\delta_{c}(\delta_{1c}) = 2SZD + \delta_{e}$$
 (3)

where  $\delta_e$  is the elastic component of CTOD. The value of  $\delta_c$  was obtained by employing experimental SZD values and computing  $\delta_e$  using

$$\delta_{\rm e} = \frac{K_1^2(1-v^2)}{2E\sigma_{\rm v}} \tag{4}$$

The  $\delta_c$  values thus obtained are given in Table V. As the SZD values used in Equation 3 are based on the mid-thickness measurement, the  $\delta_c$  values correspond to a plane strain condition and are thus the  $\delta_{lc}$  values in reality.

The  $\delta_{lc}$  values, as determined above, may be compared with the various initiation toughness values reported in Table V. It is more appropriate to compare the  $\delta_{lc}$  values with the initiation toughness determined using  $\delta_{max}$  against  $\Delta a_{max} R$  curves as these are believed to provide a plane strain condition. Table V shows  $\delta_{max}^{B}$  and  $\delta_{max}^{s}$  values also along with the  $\delta_{lc}$  for the two alloys. A comparison reveals that the  $\delta_{lc}$  has better agreement with  $\delta_{max}^{s}$  than that of  $\delta_{max}^{B}$ . In Mo steel, the  $\delta_{max}^{B}$  and  $\delta_{max}^{s}$  values are comparable and in fair agreement with  $\delta_{lc}$ . For Cr–Mo steel, however, the  $\delta^{B}_{max}$  are on the low side whereas the  $\delta^{s}_{max}$  have a good agreement. In overall comparison, thus, the  $\delta^{s}_{max}$  appears to better represent the crack initiation toughness for plane strain condition amongst the various  $\delta_{i}$  values. The initiation toughness  $\delta^{s}_{max}$  does not involve any assumptions defining a blunting line unlike  $\delta^{B}_{max}$ .

From Tables III and V, a close similarity is noted in the trend of variation of SZW, SZD, 'deformed region length', d, and the  $\delta_{\max}^s$  with temperature in both the steels. This is because the SZW and d are presumably a measure of crack tip ductility which varies with temperature and material. As the strain preceding crack initiation increases, more crack tip opening is required for crack initiation resulting in a higher value of initiation toughness,  $\delta^s_{max}$ . Simultaneously, an increased amount of stretching at the crack tip also results in enhanced grain deformation. An increase in the hardness of ferrite grains is caused due to enhanced deformation. A close similarity between the degree of straining (hardening) (Table III) and the  $\delta_{max}^{s}$ at each temperature further supports a closer link between the grain deformation during stretching and the initiation toughness.

#### 4.3. EGF method for CTOD toughness

The European Group on Fracture (EGF) has proposed a method for determining CTOD for crack initiation [10]. The  $\delta$ -R curve is obtained in this approach by fitting an equation of the form

$$\delta = A(\Delta a + D)^{\circ} \tag{5}$$

where A, D and c are constants to be evaluated. A blunting line equation used in this method is

$$\Delta a = 0.4 d_n G_{\rm eff} / \sigma_0 \tag{6}$$

where  $G_{eff}$  is the plasticity corrected strain energy release rate,  $\sigma_0$  the flow stress and  $d_n$  the factor used to account for strain hardening.

An attempt has been made to determine the CTOD initiation toughness using the above method and compare them with the values presented earlier.

The  $d_n$  in Equation 6 was taken from Shih's work [12]. The  $\sigma_0$  in Equation 6 was taken as

$$\sigma_0 = (\sigma_y + \sigma_{tu})/2 \tag{7}$$

where  $\sigma_{tu}$  is the true ultimate tensile stress. Table VI demonstrates the equations for the  $\delta$  against *R* curves based on Equation 5 for the two steels. In the other

TABLE VI Equations for  $\delta - R$  curves and initiation toughness using EGF's method

Alloy	Test temperature (°C)	δ <sub>av</sub> against Δa <sub>av</sub> (μm) (mm)	$\delta_{\max}$ against $\Delta a_{\max}$ ( $\mu$ m) (mm)	δ <sup>EGF</sup> av. (μm)	δ <sup>EGF</sup> (μm)
<b>X</b> ( )	20	$\delta_{av} =$	$\delta_{max} =$		
Mo steel	30 400	$4.21(\Delta a + 3.99)^{3.0}$ $435.6(\Delta a - 0.15)^{0.9}$	$120(\Lambda a + 0.94)^{0.58}$	293 124	128
<i>.</i>					120
Cr–Mo steel	200 400	$1325(\Delta a - 0.273)^{0.436}$ $391(\Delta a - 0.63)^{1.04}$	$1035(\Delta a - 0.383)^{0.58}$ $488(\Delta a - 0.036)^{0.678}$	307	125
	400	$331(\Delta u = 0.05)$	$488(\Delta u = 0.030)$	307	125

For other test situations the curve fitting was not successful

TABLE VII The M values in the J-CTOD relationship

Alloy	Test temperature (°C)	n	m <sub>b</sub>	m <sup>a</sup>
Mo steel	30	0.190	1.64	1.72
	200	0.212	1.47	1.60
	400	0.203	1.57	1.60
Cr–Mo	30	0.170	2.02	1.72
steel	200	0.203	1.91	1.32
	400	0.186	2.12	2.01

<sup>a</sup> m obtained at 0.06 (W - a)  $\simeq 0.6$  mm

situations, the curve fitting was unsuccessful, i.e. without convergence to give regression coefficients. The initiation toughness values,  $\delta_{av}^{EGF}$  and  $\delta_{max}^{EGF}$  using this method are reported in Table VI based on the  $\delta_{av}$ against  $\Delta a_{av}$  and the  $\delta_{max}$  against  $\Delta a_{max} R$  curves, respectively. A comparison with Table IV shows that the  $\delta_i$  values are in good agreement except for the Cr-Mo steel at 400 °C where the difference is considerable.

#### 4.4. J-CTOD relationship

The J integral values were also obtained in the alloys investigated using the standard method [8] as reported elsewhere [11]. The J-CTOD relationship is written as

$$J = m\delta\sigma_0 \tag{8}$$

where *m* is a constant. The values of *m* were obtained during blunting (designated as  $m_b$ ) and also during crack growth upto  $0.06(W - a) \simeq 0.6$  mm. The  $m_b$  and *m* values are given in Table VII. The  $m_b$  values are found to vary between 1.47 and 2.12 whereas the *m* values lie between 1.32 and 2.01 for the alloys investigated.

An attempt has been made to correlate m with the strain hardening exponent, n of the alloys as shown in Fig. 6. The figure also includes data from HAZ and weld zone of alloys taken from [11]. Superimposed on the figure are the results from other investigators [13–15]. Though, most of the experimental points from the present work fall within the scatterband suggested by literature, no clear trend between  $m(m_b)$  against n can be inferred. It has been reported [15] that m increases with n upto a limiting value of n beyond which the m remains constant. Nevertheless, it requires further investigation.



Figure 6 Variation of  $m(m_b)$  with  $n. (\emptyset \text{ Mo, } m, \text{AR}; \bigcirc \text{Cr-Mo, } m, \text{AR}; \otimes \text{Mo, } m, \text{HAZ}; * \text{Cr-Mo, } m, \text{HAZ}; \triangle \text{Cr-Mo, } m, \text{WZ}; \emptyset \text{ Mo, } M_b, \text{AR}; \emptyset \text{ Cr-Mo, } m_b, \text{AR}) (\ddagger \text{Pandey [13]}, --- \text{Robinson [14]}, --- \text{Slatcher [15]}).$ 

#### 5. Conclusions

The conclusions are as follows

1. The crack initiation toughness at 0.2 mm crack growth  $\delta^{0.2}$  is higher than the  $\delta^{s}$  and  $\delta^{B}$  initiation toughness which are based on the stretched zone width and the blunting line ( $\delta = 2\Delta a$ ) approach, respectively.

2. An initiation toughness based on stretched zone depth ( $\delta_{Ic}$ ) has good agreement with  $\delta^s_{max}$  in both the steels.

3. The crack-tip stretching is followed by elongation and hardening of ferrite grains. The length of deformed grain region (d) and degree of hardening at different temperatures have a trend similar as the initiation toughness,  $\delta^{s}$  or  $\delta_{le}$ .

4. The constant factor, m in the *J*-CTOD relationship lies between 1.3 and 2.1 and has no definite relationship with strain hardening exponent.

5. The EGF method for toughness determination is not applicable in all the situations.

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